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Three-Dimensional Inverse Estimation of Heat Generation in Board Mounted Chips

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The conjugate gradient method (CGM) and the general purpose commercial code CFX 4.2-based inverse algorithm are utilized in a three-dimensional inverse heat conduction problem to estimate the strength of heat generation for chips on a printed circuit board (PC-board). The mathematical formulations for simultaneously solving the strength of heat generation for multiple chips are derived. The advantage of calling the CFX 4.2 code as a subroutine in the present inverse calculation lies in that many difficult but practical three-dimensional inverse problems can be solved under this construction because the general-purpose commercial code has the ability to solve the direct problem easily. Results obtained by using the CGM to solve this three-dimensional inverse problem for two chips are justified based on the numerical experiments with the simulated exact and inexact measurements. It is concluded that accurate heat generation for each chip can be estimated simultaneously by the CGM except for the final time. The reason for and improvement of this singularity are addressed.

Nomenclature

C_p = heat capacity
g = unknown volumetric heat generation
h = heat transfer coefficient
J = functional defined by Eq. (2)
J' = gradient of functional defined by Eq. (11a)
k = thermal conductivity
P = direction of descent defined by Eq. (3c)
T = calculated temperature
Y = measured temperature
γ = conjugate coefficient defined by Eq. (3e)
ΔT = solution of sensitivity problem
ε = convergence criteria
λ = solution for adjoint problem
ρ = density
σ = standard deviation of the measurement errors
Ω = computational domain
ω = random number

Superscript

$\hat{}$ = estimated values

I. Introduction

THE demand for quieter, smaller, more powerful, safer, and more reliable electronics equipment has made cooling a concern for manufacturers. However, before the cooling system can be designed, the strength of heat generation for chips on a printed circuit board (PC-board) must be given or calculated.

The chip construction is very complicated, and, for this reason, it is difficult to model the thermal behavior of chip mathematically. Moreover, the chips are composed of different materials, and so the heat generation of a chip must therefore be a function of position and time.

The task now is how to obtain the strength of heat generation of the chip having so many uncertainties, as was already stated. The solution to this might be the techniques of the inverse heat conduction problem (IHCP).[1]

The direct heat conduction problems are concerned with the determination of temperature distribution in the interior of a solid when the initial and boundary conditions, thermophysical properties, and heat generation are specified. In contrast, the IHCP that is discussed here involves the determination of the strength of heat generation for chips by utilizing the measured temperature history on the back surface of a PC-board.

The IHCP has numerous applications in many fields of science and engineering. For the two-dimensional inverse problems in regular coordinates, that is, rectangular, cylindrical, and spherical coordinates, or in an irregular domain, various approaches are available and can be found in the literature.[2–6] However, three-dimensional inverse problems with an irregular domain are very limited in the literature.

Recently, Huang and Wang[7] used the conjugate gradient method and commercial code CFX 4.2 (Ref. 8) in a three-dimensional IHCP of estimating surface heat flux. In their paper, CFX 4.2 is a subroutine in the main program where the algorithm of the conjugate gradient method (CGM) is used. The bridge between CFX and CGM is the input/output file. Those files should be arranged such that their format can be recognize by CFX and CGM. CFX 4.2 is available from AEA Technology, and the method of control volumes is used to solve the thermal problems.

More recently, Huang and Chen[9] extended a similar idea to a three-dimensional IHCP of estimating surface heat flux and obtained good predictions. The objective of the present study is to utilize the CFX 4.2 code as the subroutine in solving the three-dimensional inverse problems by CGM in estimating the strength of heat generation of chips on the PC-board.

The CGM is an iterative regularization method because the regularization procedure is performed as part of the iterative processes, and, thus, the determination of optimal regularization conditions is not needed. The CGM is based on Alifanov's perturbation principle,[10] which divides the inverse problem into three subproblems: 1) the direct problem, 2) the sensitivity problem, and 3) the adjoint problem. These will be described hereafter. These three problems are also solved by CFX 4.2, and the calculated values are used in CGM for inverse calculations.

Finally, the inverse solutions for two transient heat conduction problems with different irregular geometries and different boundary conditions will be illustrated to show the validity of using the CGM in the present three-dimensional IHCP.

II. Direct Problem

To illustrate the methodology for developing expressions for use in determining the unknown heat generation of chips on the

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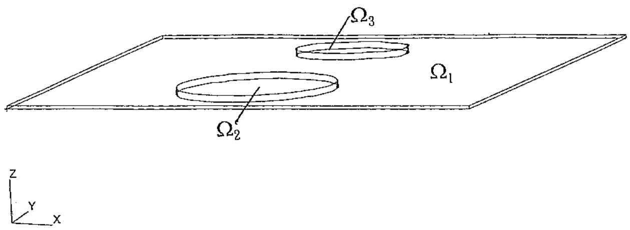

Fig. 1a Geometry and coordinates.

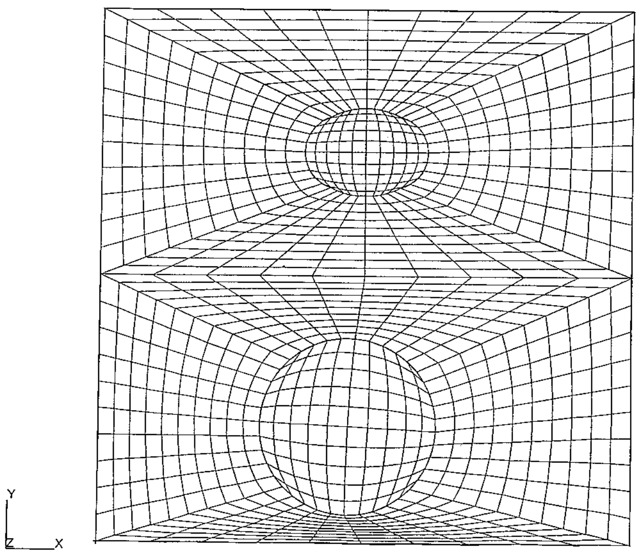

Fig. 1b Grid system for present study.

PC-board by using CGM and CFX 4.2, we consider the following three-dimensional IHCP. For a domain $\mathbf{\Omega}$, where $\{\mathbf{\Omega}\} = \{\Omega_1 \cup \Omega_2 \cup, \ldots, \cup \Omega_I\}$, Ω_1 represents the PC-board and Ω_j, $j = 2$–I are the domains for integrated circuit (IC) chips, that is, there are a total of $(I-1)$ IC chips on the PC-board. The initial temperature in $\mathbf{\Omega}$ equals T_0. When $t > 0$, the boundary conditions on all of the boundary surfaces are subjected to the Robin boundary conditions with heat transfer coefficient h and ambient temperature T_∞. It also assumed that there exists a volumetric heat generation $g(\Omega_j, t)$ in each chip Ω_j. Figure 1a shows the geometry and the coordinates (two-chips problem, that is, $I = 3$) for the three-dimensional physical problem considered here.

The mathematical formulation of the three-dimensional heat conduction problem for a PC-board with chips is given by

$$k(\Omega_i)\left[\frac{\partial^2 T(\Omega_i, t)}{\partial x^2} + \frac{\partial^2 T(\Omega_i, t)}{\partial y^2} + \frac{\partial^2 T(\Omega_i, t)}{\partial z^2}\right]$$

$$+ g(\Omega_j, t)\delta(\Omega_j - \Omega_i) = \rho(\Omega_i) C_P(\Omega_i)\frac{\partial T(\Omega_i, t)}{\partial t}, \quad \text{in} \quad \mathbf{\Omega}$$

$$i = 1\text{–}I, \qquad j = 2\text{–}I, \qquad t > 0 \quad \text{(1a)}$$

$$\pm k\frac{\partial T}{\partial n} = h(T - T_\infty) \quad \text{on every boundry surface}, \quad t > 0 \quad \text{(1b)}$$

$$T(\Omega_i, 0) = T_0, \qquad \text{in} \qquad \mathbf{\Omega}, \qquad i = 1\text{–}I, \qquad t = 0 \quad \text{(1c)}$$

The conditions on the interface between chips and PC-board are given as

$$T(\Omega_1 \cap \Omega_j) = T(\Omega_j) \quad \text{and}$$

$$k(\Omega_1 \cap \Omega_j)\frac{\partial T(\Omega_1 \cap \Omega_j, t)}{\partial n} = k(\Omega_j)\frac{\partial T(\Omega_j, t)}{\partial n}$$

$$j = 2\text{–}I, \qquad t > 0 \quad \text{(1d)}$$

The solution for the preceding three-dimensional transient heat conduction problem in domain $\mathbf{\Omega}$ is solved using CFX 4.2 and the FORTRAN subroutine USRBCS. The direct problem considered here is concerned with the determination of the medium temperature when all of the boundary conditions at all boundaries and heat generations are known.

III. Inverse Problem

For the inverse problem, the volumetric heat generation $g(\Omega_j, t)$ in Ω_j, $j = 2$–I are regarded as being unknown, but everything else in Eq. (1) is known. In addition, temperature readings using infrared scanners taken at some appropriate locations and times on the bottom surface S_b are considered available.

Let the temperature reading taken by infrared scanners on bottom surface S_b be $Y(S_b, t) \equiv Y(x_m, y_m, z_m, t) \equiv Y_m(t), m = 1$–$M$, where M represents the number of measured temperature extracting points. We note that the measured temperature $Y_m(t)$ contains measurement errors. Then the inverse problem can be stated as follows: By the utilizion of the aforementioned measured temperature data $Y_m(t)$, estimate the unknown heat generations, $g(\Omega_j, t)$, in Ω_j, $j = 2$–I.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[g(\Omega_j, t)] = \int_{t=0}^{t_f} \sum_{m=1}^{M} [T(x_m, y_m, z_m, t) - Y(x_m, y_m, z_m, t)]^2 \,\mathrm{d}t$$

$$= \int_{t=0}^{t_f} \sum_{m=1}^{M} [T_m(t) - Y_m(t)]^2 \,\mathrm{d}t \quad \text{(2)}$$

Here, $T_m(t)$ are the estimated or computed temperatures at the measured temperature extracting locations (x_m, y_m, z_m) at time t. These quantities are determined from the solution of the direct problem given earlier by using an estimated $\hat{g}(\Omega_j, t)$ for the exact $g(\Omega_j, t)$. Here the carat denotes the estimated quantities, and t_f is the final time.

IV. Conjugate Gradient Method for Minimization

The following iterative process based on the conjugate gradient method[10] is now used for the estimation of unknown heat sources $g(\Omega_j, t)$ by minimizing the functional $J[g(\Omega_j, t)]$

$$\hat{g}^{n+1}(\Omega_j, t) = \hat{g}^n(\Omega_j, t) - \beta_j^n P^n(\Omega_j, t)$$

$$\text{for} \quad j = 2\text{–}I, \qquad n = 0, 1, 2, \ldots \quad \text{(3a)}$$

or, in vector form,

$$\hat{\boldsymbol{g}}^{n+1}(t) = \hat{\boldsymbol{g}}^n(t) - \boldsymbol{\beta}^n \boldsymbol{P}^n(t) \quad \text{(3b)}$$

where β_j^n are the search step sizes in going from iteration n to iteration $n+1$ and $P^n(\Omega_j, t)$ are the directions of descent, that is, search directions, given by

$$P^n(\Omega_j, t) = J'^n(\Omega_j, t) + \gamma_j^n P^{n-1}(\Omega_j, t), \qquad \text{for} \quad j = 2\text{–}I \quad \text{(3c)}$$

In vector form this is

$$\boldsymbol{P}^{n+1}(t) = \boldsymbol{J}'^n(t) - \boldsymbol{\gamma}^n \boldsymbol{P}^{n-1}(t) \quad \text{(3d)}$$

which are the conjugations of the gradient direction $J'^n(\Omega_j, t)$ at iteration n and the direction of descent $P^{n-1}(\Omega_j, t)$ at iteration $n-1$. The conjugate coefficient is determined from

$$\gamma_j^n = \frac{\int_{t=0}^{tf} \int_{\Omega_j} (J'^n)^2 \,\mathrm{d}\Omega_j \,\mathrm{d}t}{\int_{t=0}^{tf} \int_{\Omega_j} (J'^{n-1})^2 \,\mathrm{d}\Omega_j \,\mathrm{d}t} \quad \text{(3e)}$$

with $\gamma_j^0 = 0$.

We note that when $\gamma_j^n = 0$ for any n, in Eq. (3b), the direction of descent $P^n(\Omega_j, t)$ becomes the gradient direction, that is, the steepest descent method is obtained. The convergence of the preceding iterative procedure in minimizing the functional J is guaranteed as shown in the paper by Lasdon et al.[11]

To perform the iterations according to Eq. (3a), we need to compute the step size β_j^n and the gradient of the functional $J'^n(\Omega_j, t)$. To develop expressions for the determination of these two quantities, the sensitivity problems and an adjoint problem are constructed as described hereafter.

A. Sensitivity Problem and Search Step Size

Because the problem involves $(I-1)$ unknown time-dependent volumetric heat sources, $g(\Omega_j, t)$, $j = 2$–I. To derive the sensitivity problem for each unknown heat source, we should perturb one unknown heat source at a time.

The sensitivity problems are obtained from the original direct problem defined by Eq. (1) in the following manner: It is assumed that when $g(\Omega_j, t)$ undergoes a variation $\Delta g(\Omega_j, t)\delta(i-j)$, where $\delta(\bullet)$ is the Dirac delta function and $i = 1$–I and $j = 2$–I, then $T(\Omega_i, t)$ is perturbed by $\Delta T_j(\Omega_i, t)$. Then replacing $g(\Omega_j, t)$ in the direct problem by $g(\Omega_j, t) + \Delta g(\Omega_j, t)\delta(i-j)$ and $T(\Omega_i, t)$ by $T(\Omega_i, t) + \Delta T_j(\Omega_i, t)$, subtracting from the resulting expressions the direct problem, and neglecting the second-order terms, the following $(I-1)$ sensitivity problems for the sensitivity functions $\Delta T_j(\Omega_i, t)$ are obtained:

$$k(\Omega_i)\left[\frac{\partial^2 \Delta T_j(\Omega_i, t)}{\partial x^2} + \frac{\partial^2 \Delta T_j(\Omega_i, t)}{\partial y^2} + \frac{\partial^2 \Delta T_j(\Omega_i, t)}{\partial z^2}\right] + \Delta g(\Omega_j, t)\delta(\Omega_i - \Omega_j) = \rho(\Omega_i)C_P(\Omega_i)\frac{\partial \Delta T_j(\Omega_i, t)}{\partial t}$$

$$\text{in} \quad \mathbf{\Omega}, \qquad i = 1\text{–}I, \qquad j = 2\text{–}I, \qquad t > 0 \tag{4a}$$

$$\pm k\frac{\partial \Delta T_j}{\partial n} = h\Delta T_j \quad \text{on every boundary surface}, \quad t > 0 \tag{4b}$$

$$\Delta T_j = 0, \qquad \text{in} \quad \mathbf{\Omega}, \qquad t = 0 \tag{4c}$$

The conditions on the interface between chips and PC-board are given as

$$\Delta T(\Omega_1 \cap \Omega_j) = \Delta T(\Omega_j) \quad \text{and}$$

$$k(\Omega_1 \cap \Omega_j)\frac{\partial \Delta T(\Omega_1 \cap \Omega_j, t)}{\partial n} = k(\Omega_j)\frac{\partial \Delta T(\Omega_j, t)}{\partial n}$$

$$j = 2\text{–}I, \qquad t > 0 \tag{4d}$$

The CFX 4.2 is used to solve these $(I-1)$ sensitivity problems.

The functional $J[\hat{g}^{n+1}(t)]$ for iteration $n+1$ is obtained by rewriting Eq. (2) as

$$J[\hat{\boldsymbol{g}}^{n+1}(t)] = \int_{t=0}^{t_f}\sum_{m=1}^{M}\left[T_m[\hat{\boldsymbol{g}}^n(t) - \boldsymbol{\beta}^n\boldsymbol{P}^n(t)] - Y_m\right]^2 \mathrm{d}t \tag{5}$$

where we replaced $\hat{\boldsymbol{g}}^{n+1}$ by the expression given by Eq. (3a). If temperature $T_m[\hat{\boldsymbol{g}}^n(t) - \boldsymbol{\beta}^n\boldsymbol{P}^n(t)]$ is linearized by a Taylor expansion, Eq. (5a) takes the form

$$J[\hat{\boldsymbol{g}}^{n+1}(t)] = \int_{t=0}^{t_f}\sum_{m=1}^{M}\left[T_m(\hat{\boldsymbol{g}}^n) - \sum_{j=2}^{I}\beta_j^n\Delta T_{m,j}\left(P_j^n\right) - Y_m\right]^2 \mathrm{d}t \tag{6}$$

Where $T_m(\hat{\boldsymbol{g}}^n)$ is the solution of the direct problem by using estimate $\hat{g}^n$ for exact g at (x_m, y_m, z_m) and time t. The sensitivity functions $\Delta T_{m,j}(P_j^n)$ are taken as the solutions of problem (4) at the measured temperature extracting positions (x_m, y_m, z_m) and time t by letting $\Delta\boldsymbol{g} = \boldsymbol{P}^n$ (Ref. 10). The search step sizes β_j^n can be determined by minimizing the functional given by Eq. (6) with respect to β_j^n.

B. Adjoint Problem and Gradient Equation

To obtain the adjoint problem, Eq. (1a) is multiplied by the Lagrange multiplier (or adjoint functions) $\lambda_j(\Omega_i, t)$, and the resulting expression is integrated over the correspondent space and time domains. Then the result is added to the right-hand side of Eq. (2) to yield the following expression for the functional $J[g(\Omega_j, t)]$:

$$J[g(\Omega_j, t)] = \int_{t=0}^{t_f}\int_{S_b}[T - Y]^2\delta(x - x_m)\delta(y - y_m)\times\delta(z - z_m)\,\mathrm{d}S_b\,\mathrm{d}t + \int_{t=0}^{t_f}\int_{\Omega}\lambda_j(\Omega_i)\left\{k(\Omega_i)\left[\frac{\partial^2 T(\Omega_i, t)}{\partial x^2} + \frac{\partial^2 T(\Omega_i, t)}{\partial y^2} + \frac{\partial^2 T(\Omega_i, t)}{\partial z^2}\right] + g(\Omega_j, t)\delta(\Omega_i - \Omega_j) - \rho(\Omega_i)C_P(\Omega_i)\frac{\partial T(\Omega_i, t)}{\partial t}\right\}\mathrm{d}\Omega\,\mathrm{d}t$$

$$\text{in} \quad \mathbf{\Omega}, \qquad i = 1\text{–}I, \qquad j = 2\text{–}I, \qquad t > 0 \tag{7}$$

The variation ΔJ_j is obtained by perturbing $g(\Omega_j, t)$ by $\Delta g(\Omega_j, t)\delta(i-j)$ and $T(\Omega_i, t)$ by $\Delta T_j(\Omega_i, t)$ in Eq. (7), where $i = 1$–I and $j = 2$–I, subtracting from the resulting expression the original Eq. (7), and neglecting the second-order terms. We, thus, find

$$\Delta J_j = \int_{t=0}^{t_f}\int_{S_b}2(T - Y)\Delta T\delta(x - x_m)\delta(y - y_m)\times\delta(z - z_m)\,\mathrm{d}S_b\,\mathrm{d}t + \int_{t=0}^{t_f}\int_{\Omega}\lambda_j(\Omega_i)\left\{k(\Omega_i)\left[\frac{\partial^2 \Delta T(\Omega_i, t)}{\partial x^2} + \frac{\partial^2 \Delta T(\Omega_i, t)}{\partial y^2} + \frac{\partial^2 \Delta T(\Omega_i, t)}{\partial z^2}\right] + \Delta g(\Omega_j, t)\delta(\Omega_i - \Omega_j) - \rho(\Omega_i)C_P(\Omega_i)\frac{\partial \Delta T(\Omega_i, t)}{\partial t}\right\}\mathrm{d}\Omega\,\mathrm{d}t$$

$$\text{in} \quad \mathbf{\Omega}, \qquad i = 1\text{–}I, \qquad j = 2\text{–}I, \qquad t > 0 \tag{8}$$

where $\delta(\bullet)$ is the Dirac delta function and (x_m, y_m, z_m), $m = 1$–M, refer to the measured temperature extracting positions. In Eq. (8), the domain integral term is reformulated based on Green's second identity; the boundary conditions of the sensitivity problem given by Eqs. (4b) and (4c) are utilized. Finally, we found that the equations for adjoint problems are all identical to each other for $j = 2$–I. For this reason, the subscript j can be neglected, and we obtained the following adjoint problems $\lambda(\Omega_i, t)$:

$$k(\Omega_i)\left[\frac{\partial^2 \lambda(\Omega_i, t)}{\partial x^2} + \frac{\partial^2 \lambda(\Omega_i, t)}{\partial y^2} + \frac{\partial^2 \lambda(\Omega_i, t)}{\partial z^2}\right] + \rho(\Omega_i)C_P(\Omega_i)\frac{\partial \lambda(\Omega_i, t)}{\partial t} = 0$$

$$\text{in} \quad \mathbf{\Omega}, \qquad i = 1\text{–}I, \qquad t > 0 \tag{9a}$$

$$\pm k\frac{\partial \lambda}{\partial n} = h\lambda \quad \text{on every boundary surface (except for } S_b\text{)}, \quad t > 0 \tag{9b}$$

$$k\frac{\partial \lambda}{\partial n} = h\lambda + 2(T - Y)\delta(x - x_m)(y - y_m)(z - z_m)$$

$$\text{on} \quad S_b, \quad t > 0 \tag{9c}$$

$$\lambda = 0, \qquad \text{in} \quad \mathbf{\Omega}, \qquad t = t_f \tag{9d}$$

Moreover, the conditions on the interface between chips and PC-board are given as

$$\lambda(\Omega_1 \cap \Omega_j) = \lambda(\Omega_j)$$

$$k(\Omega_1 \cap \Omega_j)\frac{\partial\lambda(\Omega_1 \cap \Omega_j, t)}{\partial n} = k(\Omega_j)\frac{\partial\lambda(\Omega_j, t)}{\partial n}$$

$$j = 2\text{–}I, \qquad t > 0 \qquad \text{(9e)}$$

The adjoint problem is different from the standard initial value problems in that the final time conditions at time $t = t_f$ is specified instead of the customary initial condition. However, this problem can be transformed to an initial value problem by the transformation of the time variables as $\tau = t_f - t$. Then CFX can be used to solve the preceding adjoint problem.

Finally, the following integral term is left:

$$\Delta J_j = \int_{t=0}^{t_f}\int_{\Omega_j} \lambda(\Omega_j, t)\Delta g(\Omega_j, t)\,\mathrm{d}\Omega_j\,\mathrm{d}t \qquad \text{(10a)}$$

By definition,[10] the functional increment can be presented as

$$\Delta J_j = \int_{t=0}^{t_f}\int_{\Omega_j} J'[g(\Omega_j, t)]\Delta g(\Omega_j, t)\,\mathrm{d}\Omega_j\,\mathrm{d}t \qquad \text{(10b)}$$

A comparison of Eqs. (10a) and (10b) leads to the following expression for the gradient of the functional $J[g(\Omega_j, t)]$:

$$J'[g(\Omega_j, t)] = \lambda(\Omega_j, t)|_{\text{in}\Omega_j} \qquad \text{(11a)}$$

We note that the gradient J' at final time $t = t_f$ is always equal to zero because $\lambda(\Omega_j, t_f) = 0$. If the initial guessed values of g^0 are incompatible with this constraint on the gradients at the final time, then the estimated strength of heat generation of the chips will deviate from their exact values near the final time. In fact, this occurs in the numerical test cases to be presented hereafter. However, for those test cases this singularity at $t = t_f$ can be avoided by using the artificial end conditions

$$J'(\Omega_j, t_f) = \lambda(\Omega_j, t_f - \Delta t) \qquad \text{(11b)}$$

Here, Δt denotes the time increment used in CFX.

When the artificial gradient equation (11b) is replaced with the gradient equation (11a), the singularity at final time $t = t_f$ can be avoided in the present study and a reliable inverse solution can be obtained.

C. Stopping Criterion

If the problem contains no measurement errors, the traditional check condition is specified as

$$J\left[\hat{g}^{n+1}(\Omega_j, t)\right] < \varepsilon \qquad \text{(12a)}$$

where ε is a small specified number. However, the observed temperature data may contain measurement errors. Therefore, we do not expect the functional equation (2) to be equal to zero at the final iteration step. Following Refs. 7, 9, and 10, we use the discrepancy principle as the stopping criterion, that is, we assume that the temperature residuals may be approximated by

$$T_m(t) - Y_m(t) \approx \sigma \qquad \text{(12b)}$$

where σ is the standard deviation of the measurements, which is assumed to be a constant. When Eq. (12b) is substituted into Eq. (2), the following expression is obtained for stopping criteria ε:

$$\varepsilon = M\sigma^2 t_f \qquad \text{(12c)}$$

Then, the stopping criterion is given by Eq. (12a) with ε determined from Eq. (12c).

V. Computational Procedure

The computational procedure for the solution of this inverse problem using CGM may be summarized as follows:

Suppose $\hat{g}^n(\Omega_j, t)$ is available at iteration n.

1) Solve the direct problem given by Eq. (1) for $T(\Omega_i, t)$.

2) Examine the stopping criterion given by Eq. (12a) with ε given by Eq. (12c). Continue if not satisfied.

3) Solve the adjoint problem given by Eq. (9) for $\lambda(\Omega_i, t)$.

4) Compute the gradient of the functional J' from Eq. (11).

5) Compute the conjugate coefficient γ_j^n and direction of descent $P^n(\Omega_j, t)$ from Eqs. (3c) and (3b), respectively.

6) Set $\Delta g(\Omega_j, t) = P^n(\Omega_j, t)$, and solve the sensitivity problem given by Eq. (4) for $\Delta T(\Omega_i, t)$.

7) Compute the search step size β_j^n from Eq. (6).

8) Compute the new estimation for $\hat{g}^{n+1}(\Omega_j, t)$ from Eq. (3a) and return to step 1.

VI. Results and Discussion

The objective of this paper is to show the validity of the CGM in simultaneously estimating the strength of the heat generation for chips $g(\Omega_j, t)$ in Ω_j accurately and with no prior information on the functional form of the unknown quantities.

To illustrate the accuracy of the CGM in predicting $g(\Omega_j, t)$ in a domain Ω with three-dimensional inverse analysis from the knowledge of transient temperature recordings, a specific two-chips example having a different form of heat generation for each chip is considered here.

To compare the results for situations involving random measurement errors, we assume normally distributed uncorrelated errors with zero mean and constant standard deviation. The simulated inexact measurement data $\boldsymbol{Y}$ can be expressed as

$$\boldsymbol{Y} = \boldsymbol{Y}_{\text{exact}} + \omega\sigma \qquad \text{(13)}$$

where $\boldsymbol{Y}_{\text{exact}}$ is the solution of the direct problem with an exact heat generation $g(\Omega_j, t)$, σ is the standard deviation of the measurements, and ω is a random variable that is generated by subroutine DRNNOR of the IMSL[12] and will be within -2.576–2.576 for a 99% confidence bound. In other words, we may say the measurement uncertainty is 100 : 1.

One of the advantages of using the CGM to solve the inverse problems is that the initial guesses of the unknown quantities can be chosen arbitrarily. In all of the test cases considered here, the initial guesses of $\hat{g}(\Omega_j, t)$ are taken as $\hat{g}(\Omega_j, t)_{\text{initial}} = 0.0$. We now present the numerical experiments in determining $g(\Omega_j, t)$ by the inverse analysis using the CGM.

VII. Numerical Test Case

The geometry for numerical test case is shown in Fig. 1a, which represents a PC-board (Ω_1) where one circular (Ω_2) chip and one elliptical (Ω_3) chip are mounted on the surface, that is, $I = 3$ for the present case. Because the thickness of chips is very thin, we will focus on only the thin wall problem and assume heat generation is a function of time t and surface positions x and y only (not function of z), that is, $g(\Omega_j, t) = g(x, y, t) = g(I, J, t)$.

For the numerical experiments considered here, the boundary conditions on all of the boundaries are subjected to the Robin type (convection) boundary condition, the grids along the x, y, and z directions for Ω_1, Ω_2, and Ω_3 are taken as $30 \times 30 \times 2$, $10 \times 10 \times 3$, and $10 \times 10 \times 3$, respectively. The time interval is chosen as 20, that is, $t_f = 20$, and time step $\Delta t = 1$ is used. Therefore, a total of 4000 unknown discrete heat generations are to be determined in the present study. The initial temperature T_0 and ambient temperature T_∞ are all taken as 0. The thermal properties k, ρ, C_p, and h are all assumed equal to 3 for Ω_1, 2 for Ω_2, and 1 for Ω_3. The measured temperature-extracting positions are the same as the grid points on the board surface. The number of measured temperatures M by using infrared scanners is thus taken as 900. The grid system for the present study is shown in Fig. 1b.

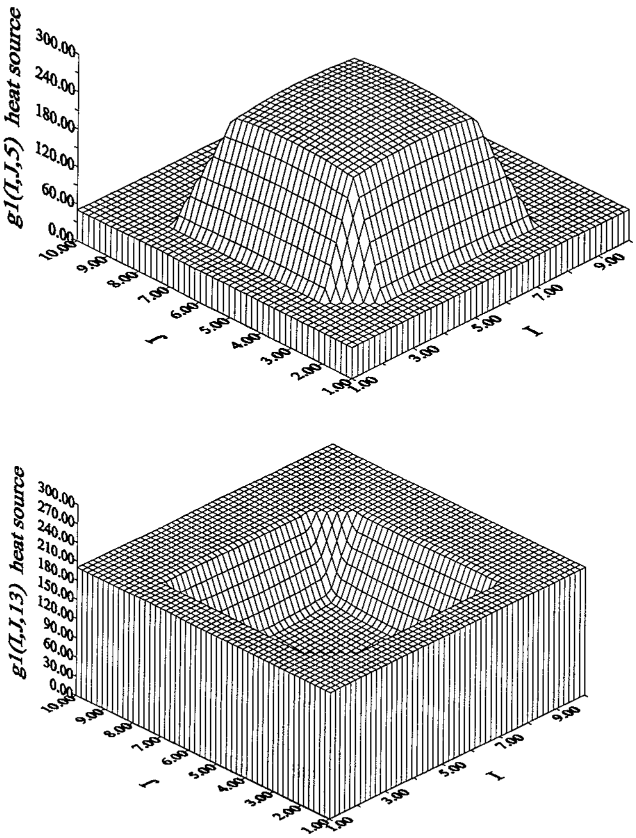

Fig. 2 Exact heat source $g_1(I, J, t)$ at $t = 5$ and 13.

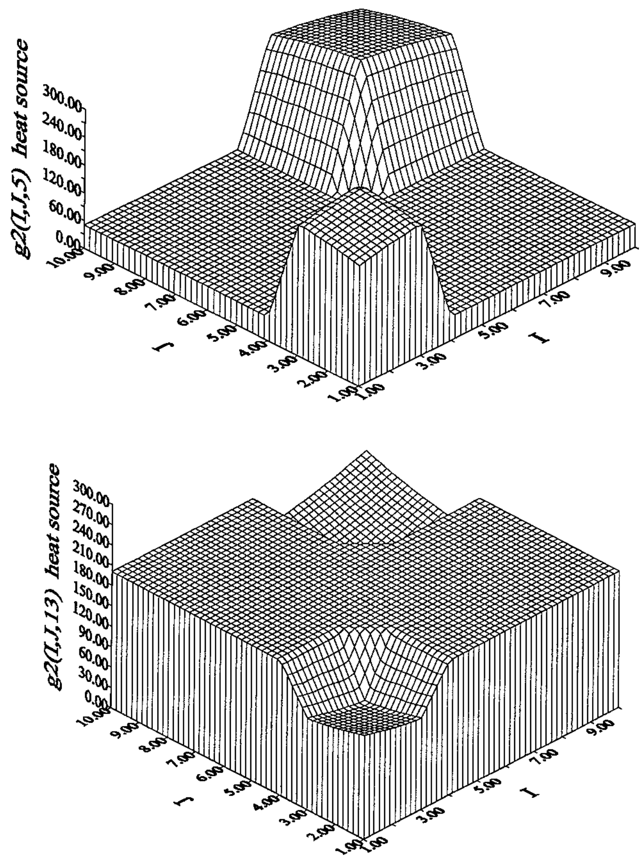

Fig. 3 Exact heat source $g_2(I, J, t)$ at $t = 5$ and 13.

The exact strength of the unknown transient heat generation $g(\Omega_2, t) = g_1(I, J, t)$ in the first chip Ω_2 is assumed as

$$g_1(I, J, t) = 2 \times \text{abs}\{[20 - (5 - J)^2 - (5 - I)^2] \times \sin[(t/10)\pi] + 100\}, \quad \begin{cases} 3 \le I \le 7 \\ 3 \le J \le 7, \end{cases} \quad 0 < t \le 8$$

$$g_1(I, J, t) = 50, \text{ elsewhere}$$

$$g_1(I, J, t) = 2 \times \text{abs}\{[17 - (5 - J)^2 - (5 - I)^2] \times \sin[(t/10)\pi] + 50\}, \quad \begin{cases} 3 \le I \le 7 \\ 3 \le J \le 7, \end{cases} \quad 8 < t \le 20$$

$$g_1(I, J, t) = 100, \text{ elsewhere} \tag{14a}$$

and the strength of the unknown transient heat generation $g(\Omega_3, t) = g_2(I, J, t)$ in the second chip Ω_3 is assumed as

$$g_2(I, J, t) = 2 \times \text{abs}\{[20 - (5 - J)^2 - (5 - I)^2] \times \sin[(t/10)\pi] + 100\}, \quad \begin{cases} 3 \le I \le 7 \\ 3 \le J \le 7, \end{cases} \quad 0 < t \le 8$$

$$g_2(I, J, t) = 50, \text{ elsewhere}$$

$$g_2(I, J, t) = 2 \times \text{abs}\{[17 - (5 - J)^2 - (5 - I)^2] \times \sin[(t/10)\pi] + 50\}, \quad \begin{cases} 3 \le I \le 7 \\ 3 \le J \le 7, \end{cases} \quad 8 < t \le 20$$

$$g_2(I, J, t) = 200, \text{ elsewhere} \tag{14b}$$

where I and J represent the grid index for a chip on the x–y plane. The exact plots for $g_1(I, J, t)$ and $g_2(I, J, t)$ at $t = 5$ and $t = 13$ are shown in Figs. 2 and 3, respectively.

Note that in the present test case we use $\hat{g}(\Omega_j, t)_{\text{initial}} = 0.0$, but now $g(\Omega_j, t_f) \neq 0$; therefore, we conclude that the singularity at final time t_f will happen in this case and the modified gradient at final time in Eq. (11b) must be used to overcome this singularity. The inverse solutions near final time under this consideration are very accurate. In this test case the estimated $\hat{g}(\Omega_j, t)$ is chosen up to $t = 19$.

The inverse analysis is first performed by assuming exact measurements, $\sigma = 0.0$. The estimated $g_1(I, J, t)$ and $g_2(I, J, t)$ after 30 iterations at $t = 5$ and 13 is shown in Figs. 4 and 5, respectively. It can be seen from Figs. 2 and 4 and 3 and 5 that the estimations are very accurate. The average errors for $g_1(I, J, t)$ and $g_2(I, J, t)$ are calculated as 0.25 and 0.28%, respectively, where the average error for the estimated heat generation is defined as

$$\text{average error\%} = \left[\sum_{I=1}^{10} \sum_{J=1}^{10} \sum_{t=1}^{t_f} \left| \frac{g_j(I, J, t) - \hat{g}_j(I, J, t)}{g_j(I, J, t)} \right| \right] \div (I \times J \times t) \times 100\%, \qquad j = 1, \quad 2 \tag{15}$$

Here, I and J represent the index of discreted unknown heat generation in Ω_2 and Ω_3, whereas t denotes the index of discretized time. Here, $g_j(I, J, t)$ and $\hat{g}_j(I, J, t)$ denote the exact and estimated values of the heat source.

Next, let us discuss the influence of the measurement errors on the inverse solutions. First, the measurement error for the temperatures measured by infrared scanners is taken as $\sigma = 0.87$ (about 2% of the average measured temperature), then the error is increased to $\sigma = 2.18$ (about 5% of the average measured temperature). The estimated $g_1(I, J, t)$ and $g_2(I, J, t)$ at $t = 5$ and 13 for these two test cases are shown in Figs. 6–9. The number of iterations for $\sigma = 0.87$ case is only 12, and the average errors for $g_1(I, J, t)$ and $g_2(I, J, t)$ are calculated as 2.8 and 2.3%, respectively. For the $\sigma = 2.18$ case, the number of iterations is only six, and the average errors for $g_1(I, J, t)$ and $g_2(I, J, t)$ are calculated as 6.1 and 5.0%, respectively. This implies that reliable inverse solutions can still be obtained when measurement errors are considered.

To show the estimated inverse solutions more clearly, we plot Fig. 10, which is the estimated $g_1(I, J, t)$ at $t = 5$ and 13 obtained

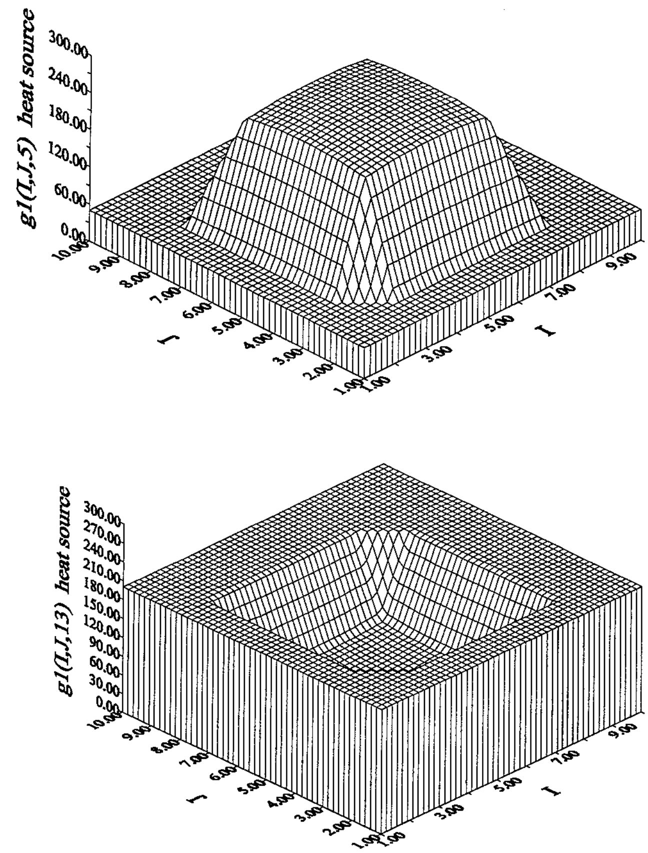

Fig. 4 Estimated heat source $g_1(I, J, t)$ at $t = 5$ and 13 using $\sigma = 0.0$.

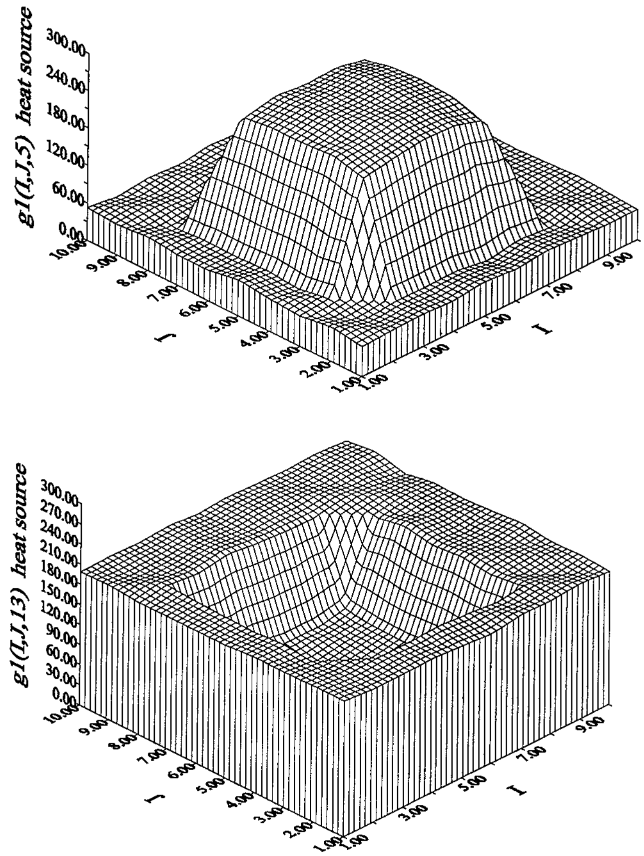

Fig. 6 Estimated heat source $g_1(I, J, t)$ at $t = 5$ and 13 using $\sigma = 0.87$.

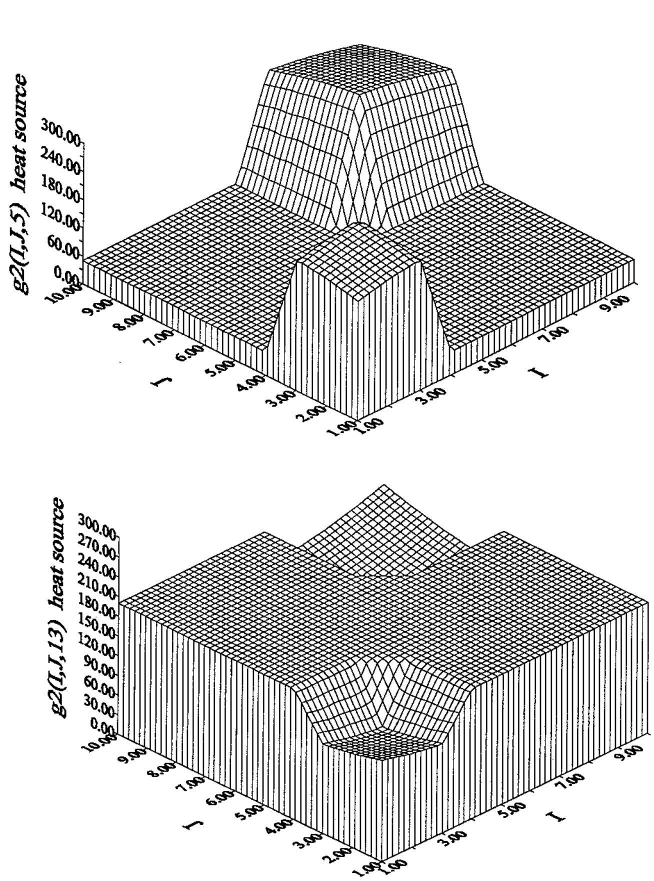

Fig. 5 Estimated heat source $g_2(I, J, t)$ at $t = 5$ and 13 using $\sigma = 0.0$.

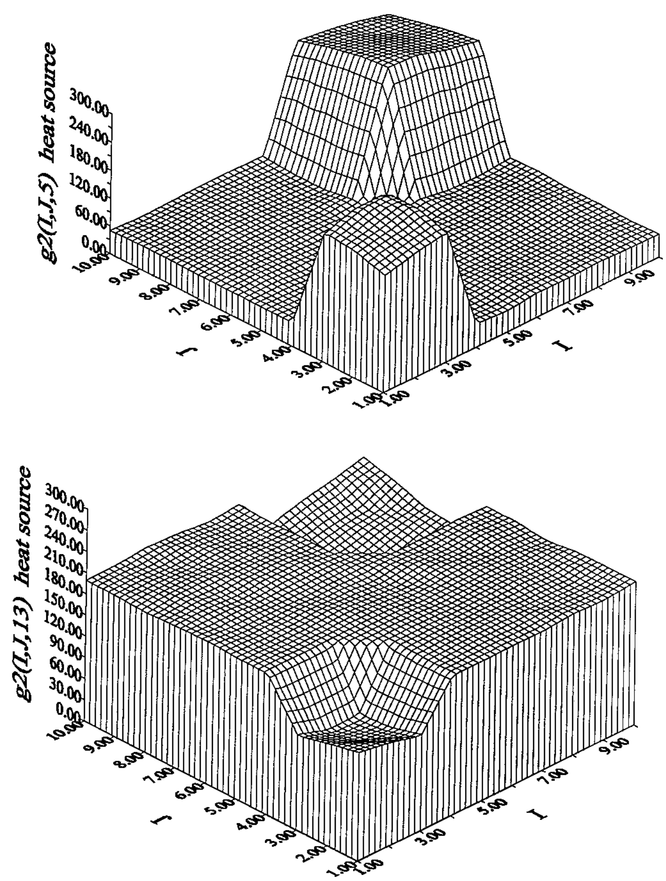

Fig. 7 Estimated heat source $g_2(I, J, t)$ at $t = 5$ and 13 using $\sigma = 0.87$.

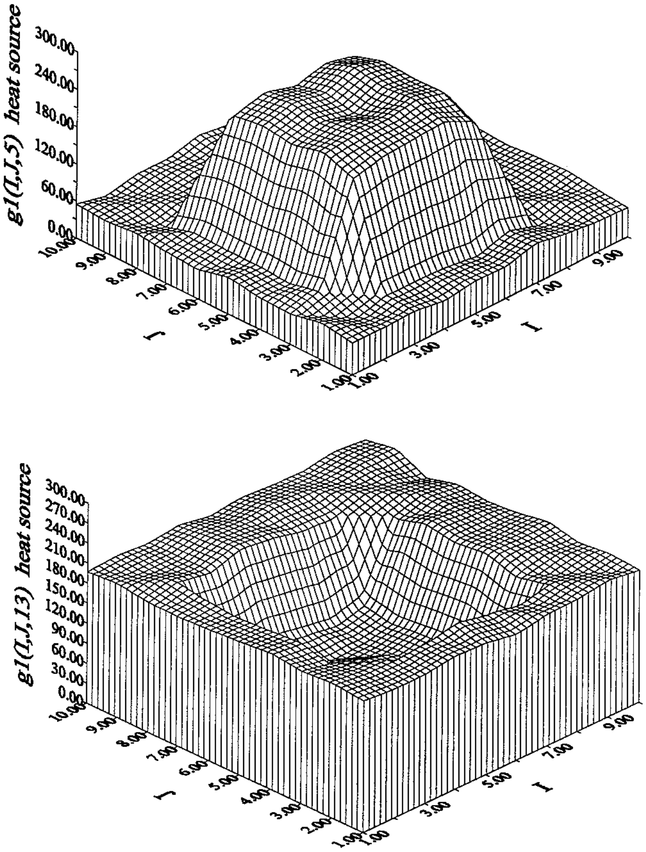

Fig. 8 Estimated heat source $g_1(I, J, t)$ at $t = 5$ and 13 using $\sigma = 2.27$.

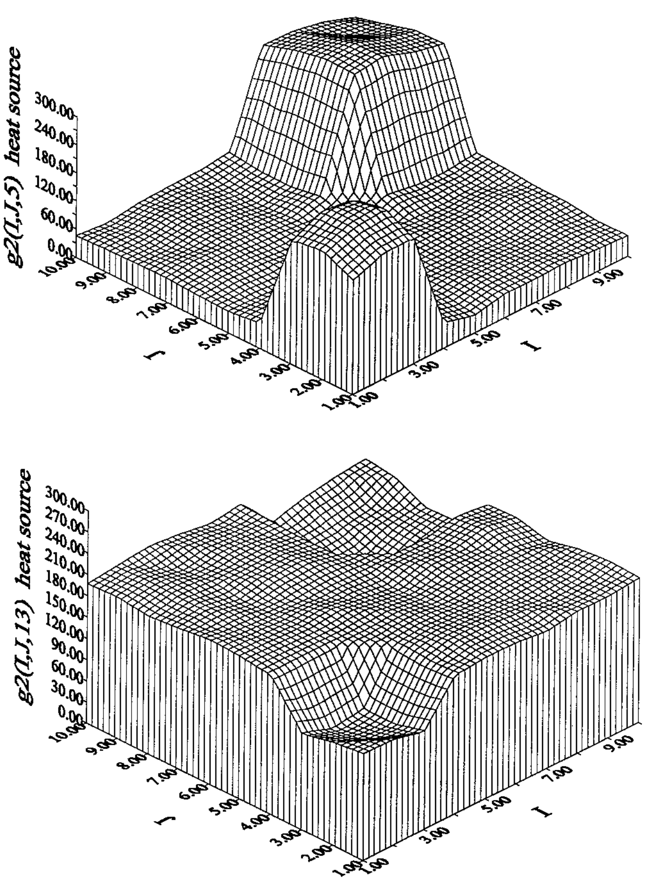

Fig. 9 Estimated heat source $g_2(I, J, t)$ at $t = 5$ and 13 using $\sigma = 2.27$.

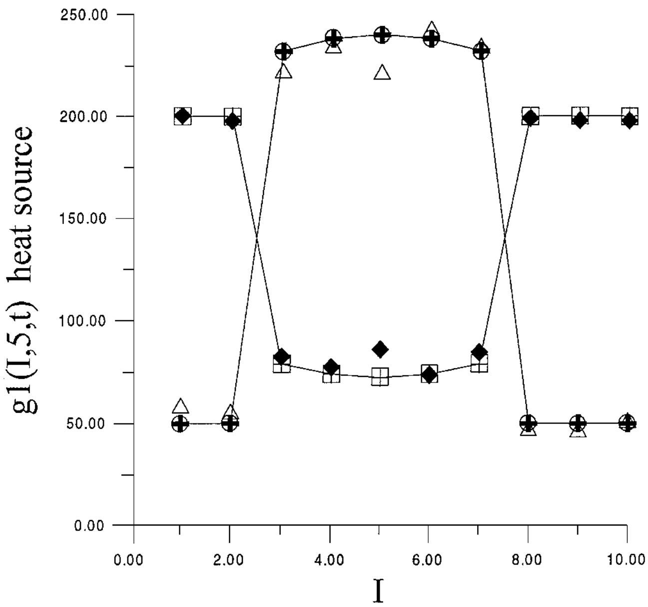

Fig. 10 Estimated heat source $g_1(I, 5, t)$ extracting from Figs. 2, 4, and 8 at $t = 5$ and 13 using $J = 4$: —✚—, exact ($t = 5$, obtained from fig 2); ○, CGM ($t = 5$, obtained from fig 4); △, CGM ($t = 5$, obtained from fig 8); —┼—, exact ($t = 13$, obtained from fig 2); □, CGM ($t = 13$, obtained from fig 4); and ◆, CGM ($t = 13$, obtained from fig 8).

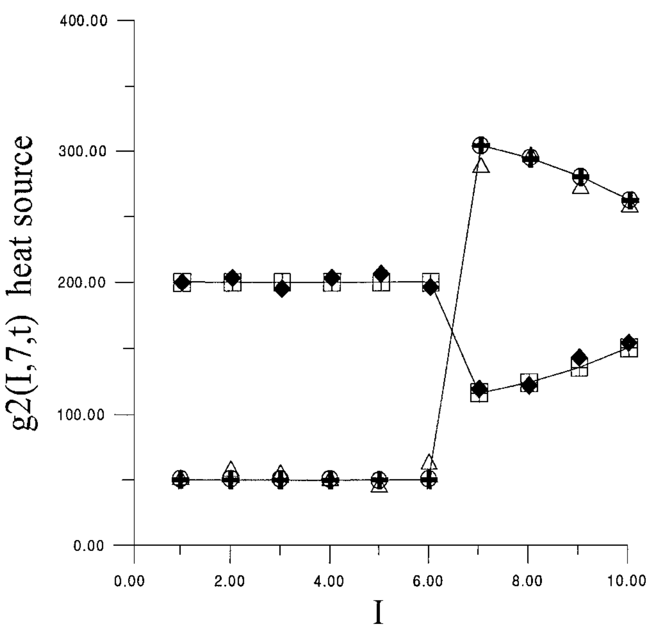

Fig. 11 Estimated heat source $g_2(I, 7, t)$ extracting from Figs. 3, 5, and 9 at $t = 5$ and 13 using $J = 7$: —✚—, exact ($t = 5$, obtained from fig 3); ○, CGM ($t = 5$, obtained from fig 5); △, CGM ($t = 5$, obtained from fig 9); —┼—, exact ($t = 13$, obtained from fig 3); □, CGM ($t = 13$, obtained from fig 5); and ◆, CGM ($t = 13$, obtained from fig 9).

from Figs. 2, 4, and 8 at $J = 5$. In Fig. 11, the estimated $g_2(I, J, t)$ at $t = 5$ and 13 obtained from Figs. 3, 5, and 9 at $J = 7$ is plotted.

From the preceding test cases, we learned that a three-dimensional IHCP in estimating the strength of the volumetric heat generation of chips on the PC-board is now completed. Reliable estimations can be obtained when using either exact or error measurements with uncertainty.

VIII. Conclusions

The CGM, along with CFX 4.2, was successfully applied for the solution of the three-dimensional IHCP to estimate simultaneously the strength of heat generation for chips on a PC-board by utilizing simulated temperature readings obtained on the board surface from infrared scanners. Several test cases involving different measurement errors were considered. The results show that the inverse solutions obtained by CGM are still very accurate as the measurement errors are increased.

From the numerical test cases in the present study we concluded that the use of CFX 4.2 as the subroutine in the three-dimensional inverse problem in estimating the strength of heat generation for chips with the CGM was successful. By the use of the same algorithm, many practical but difficult three-dimensional inverse problems can also be solved.

Acknowledgment

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